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13. ABSTRACT (Maximum 200 words)

This project concerns the development of adaptive control of linear and nonlinear dynamic system in the presence of unknown time-delays. Over the past three years, we have used the following three approaches to make inroads into this very difficult problem. These include:

- (A1) Use Lyapunov-Krasavskii functionals to develop controller structure and adaptive laws that will guarantee a stable adaptive system when the delay is unknown.
- (A2) Represent time-delay as an unknown parameter and treat the underlying system as being nonlinearly parameterized, and develop adaptive methods for nonlinearly parameterized systems.
- (A3) Develop adaptive methods in the presence of uncertainties due to failures and actuator saturation so that they can be integrated into the solutions using either of the above two approaches.

The results obtained are summarized in the final report, three technical reports, and several conference publications.

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### Appendix I

#### Approach (A1): The Lyapunov-Krasavoskii Functional

#### Approach (A2) Treating Time-delay as a nonlinear parameter

(A2)-1: A hierarchical discretized-parameter polynomial adaptive estimator for nonlinearly parameterized systems (separate attachment)

(A2)-2: A Polynomial Adaptive Controller for Nonlinearly Parameterized Systems (separate attachment)

(A2)-3: A Polynomial Adaptive Controller for Nonlinear Systems with Unknown Time Delays (separate attachment)

## 1. Introduction

Design, development, testing, and evaluation of complex systems are predicated on a thorough study that combines analysis and synthesis. As systems become more intricate, inter-disciplinary, and distributed, increasing demands are placed for the development of rigorous mathematical methods pertaining to analysis and modeling issues that arise in military science, engineering, and operations. Solutions to specific problems are invariably dependent on the state-of-the-art in mathematical science subdisciplines. Either in distributed command, control, and communications, or in guidance and control of complex semi-automated and automated systems, several problems arise related to modeling, analysis, and design of complex real-time systems. Decisions related to sensing and control have to be made under stringent performance requirements and uncertain environmental conditions. These warrant a rigorous analytical investigation of controllers that have the ability to generate compensating actions that are based on complex realistic models of the underlying system and adapt to the varying environmental characteristics and deliver a performance that meets the desired specifications. That is, a general adaptive control theory that addresses anomalies and complexities that occur in practice and generate adequate performance is crucial. A particular complexity that represents the nemesis of all control engineers is a pure time-delay and is the focus of our proposed study.

Delay systems represent a class of infinite-dimensional systems where mechanisms related to transport, propagation, or other effects related to a significant time-lag are present. Time-delays are present in almost all physical systems, simply due to the fact that there is always a delay between the application of the control input and the response of the key variables, with the actual value dependent on the underlying physics. Because of their effect on the stability properties of the resulting closed-loop system, time-delay is a particularly challenging feature in a control design. Small delays may destabilize some systems, while large delays may stabilize others. Depending on the complexity of the dynamics of the underlying system, as the delay increases, the property of the closed-loop system may switch from stability to instability and back an

infinite number of times. While in some applications, the presence of small delays raises several questions related to stability and robustness, in others, the presence of delays significantly improves the same properties. The problem of stability and stabilization becomes even more compounded if these time-delays are unknown. In problems related to fluid-flow, since time-delays are due to transport lag, any uncertainty in fluid flow velocity directly results in uncertainties in the delay. In problems related to control of distributed networks, quite often, decisions have to be made based on the actual time-delays in the system. In the absence of accurate information regarding the delays, estimates based on average delay values are made which in turn may lead to compromises in the performance. In both of these problems, adaptive methods that allow on-line estimation of time-delays using all available off-line and on-line information will prove to be useful. This proposal concerns the development of adaptive control of linear and nonlinear dynamic systems in the presence of unknown time-delays.

The field of adaptive control has addressed the problem of various kinds of uncertainties. Much of the progress in this field over the past thirty years is concerned with problems where the uncertainty takes the form of linearly occurring parameters. Recently, this form has been extended to the case when parameters occur in general nonlinear forms. Examples of dynamic systems where these nonlinearities occur include friction dynamics which include the Stribeck parameter, chemical reactors with reaction constant, magnetic bearings with air-gap as the underlying nonlinear parameter, and fed-batch fermentation processes where growth constants of enzymes affect the dynamics in a nonlinear manner. Adaptive control actions have been generated by combining an estimation of the parameters with a compensating control input that seeks to modify the dynamics of the process.

Yet another form of uncertainty that represents a significant departure from these structure is one where the time-delay  $\tau$  is unknown. While  $\tau$  can indeed be viewed as a parameter, since it directly affects the support set, none of the traditional approaches of parameter adjustment is applicable to this case. Very few results currently exist for the adaptive control of and global stabilization in linear and nonlinear systems with unknown time-delays. The goal of this project is to develop such new tools for the control of general linear and nonlinear systems with unknown time-delays.

## 2. Summary of Results

Over the past three years, we have used the following three approaches to make inroads into this very difficult problem. These include:

- (A1) Use Lyapunov-Krasavoskii functionals to develop controller structure and adaptive laws that will guarantee a stable adaptive system when the delay is unknown.
- (A2) Represent time-delay as an unknown parameter and treat the underlying system as being nonlinearly parameterized, and develop adaptive methods for nonlinearly parameterized systems.
- (A3) Develop adaptive methods in the presence of uncertainties due to failures and actuator saturation so that they can be integrated into the solutions using either of the above two approaches.

We have obtained results using each of these above approaches, and are summarized below.

### 2.1. *Approach (A1): The Lyapunov-Krasavoskii Functional*

The first system examined has time-delays in state variables ( $\tau_1$ ) and control input ( $\tau_2$ ), but  $\tau_1 - \tau_2 = \bar{\tau} > 0$ . The simplest 1<sup>st</sup> order system satisfying this condition is given by the following equation.

$$\dot{y}(t) = -\alpha y(t - \tau_1) + u(t - \tau_2),$$

where  $\alpha$  is known, but  $\tau_i$ 's are unknown.

The control structure is chosen to be

$$u(t) = -ky(t - \hat{\tau}(t)),$$

$$\dot{\hat{\tau}}(t) = -ky(t - \hat{\tau}(t))y(t) - \beta^2 y^2(t - \hat{\tau}(t)).$$

This controller is shown to be stable by using a Lyapunov-Krasovskii functional  $V$ ,

$$V(t) = \frac{1}{2} z(t)^2 + |\hat{\tau}(t)| + \varepsilon_1 \int_{-\tau}^0 \int_{t+\theta}^t y(\xi)^2 d\xi d\theta + \varepsilon_2 \int_{-\tau}^0 \int_{t+\theta}^t y(\xi - \hat{\tau}(\xi))^2 d\xi d\theta,$$

where

$$z(t) = y(t) - \alpha \int_{t-\tau_1}^t y(\theta) d\theta - k \int_{t-\tau_2}^t y(\theta - \hat{\tau}(\theta)) d\theta .$$

The details are illustrated in Appendix I.

## **2.2.      *Approach (A2): Adaptive control of systems with unknown time-delay***

An alternate approach for the control of systems with an unknown time-delay is to treat them as a special class of nonlinearly parameterized systems. While adaptive control for linearly parameterized systems is well known and has been documented in several textbooks, adaptive control of systems where parameters occur nonlinearly has begun to be investigated only recently. In order to make some inroads, a stability framework has been established for studying estimation and control of nonlinearly parameterized systems in [1]. The major stumbling block in the extension of the approaches that have been developed for linearly parameterized (LP) systems to NP systems is the inadequacy of a gradient algorithm for the control of a NP system since the underlying cost function is nonconvex. A new approach that we have successfully developed is a Polynomial Adaptive Controller (PAC) which uses a higher-order polynomial of the parameter errors to construct the Lyapunov function instead of the quadratic forms. New adaptive laws are constructed to guarantee the stability. With the new obtained freedom by adopting a higher order polynomial function, the PAC can deal with piece-wise linearly parameterized systems exactly the same as the current available adaptive controllers for linearly parameterized ones. This in turn allows us to develop a stabilizing adaptive controllers for systems with unknown delays. In Appendix I, we present adaptive controllers for two classes of nonlinearly parameterized system, with the latter including delays which are treated as unknown nonlinear “parameters”.

### 2.3. ***Approach (A3): Adaptive control in the presence of failures and actuator saturation***

A reconfigurable controller is one that automatically redesigns control laws so as to restore nominal control of a plant in the event of actuator failure or other unforeseen changes in the plant dynamics. Reconfigurable controllers have the potential to provide a significant benefit in safety in several applications. It is estimated that 14% of all fatal aircraft failures could have been prevented, in principle, by an effective reconfigurable flight control system [2]. Reconfigurable control is crucial to the development of Unmanned Air Vehicles (UAV's) as well, since UAV's are required to follow navigational commands in the presence of uncertain, possibly failure-related, disturbances. We explore how adaptive control can be applied for reconfiguration. The approach developed is applicable to any dynamic system in risk of actuator failure. It has an immediate application to fixed-wing aircraft and rotor-craft. It has the potential of increasing safety and fault tolerance in non-combat situations, and survivability in battle by adapting to battle damage.

The problem formulation is as follows. Given a failed system of the form

$$\dot{x} = A_p x + B_p \Lambda \text{sat}(u) + B_p f$$

where  $A_p$  represents the unknown failed dynamics,  $B_p$  known control effectiveness,  $\Lambda$  unknown diagonal actuator failure matrix,  $f$  unknown constant disturbance vector,  $\text{sat}(\cdot)$  is multi-dimensional saturation function, and a desired dynamics of the form

$$\dot{x}_m = A_m x_m + B_m r,$$

where  $r$  is an arbitrary bounded reference input, determine  $u$  so that all the controller's states remain bounded and  $(x - x_m)$  remains as small as possible. It is assumed that ideal gains  $K_x^*, K_r^*, k_f^*$  exist such that

$$A_p + B_p \Lambda K_x^* = A_m, \quad B_p \Lambda K_r^* = B_m, \quad B_p (\Lambda k_f^* - f) = 0$$

It is also assumed that the failed plant is controllable (the controllability matrix is full rank) and that the initial conditions lie within certain bounds, which are determined by the saturation limits and the system dynamics.



A controller of the form

$$u_c = K_x x + K_r r + k_f,$$

is proposed with adaptive laws of the form:

$$\dot{K}_x = -\Gamma_1 B_p^T P e_u x^T, \quad \dot{K}_r = -\Gamma_2 B_p^T P e_u r^T, \quad \dot{f} = -\Gamma_3 B_p^T P e_u, \quad \text{and} \quad \dot{\hat{\lambda}} = \Gamma_4 \text{diag}(\Delta u) B_p^T P e_u$$

where  $e_u = e - e_\Delta$ ,  $e = x - x_m$ ,  $A_m^T P + P A_m = -Q < 0$ , and  $\Gamma_i$  is symmetric positive definite for  $i = 1, \dots, 4$ .

Saturation is compensated for with an auxiliary error generated as

$$\dot{e}_\Delta = A_m e_\Delta + B_p \text{diag}(\hat{\lambda}) \Delta u,$$

where  $\Delta u = \text{sat}(u) - u$ , and  $\text{sat}(\cdot)$  is a multi-dimensional saturation function. The results in [3] (also in A2-4.pdf) show that  $K_x$ ,  $K_r$ ,  $k_f$ ,  $\hat{\lambda}$ , and  $e_u$  are bounded and that  $x_{\max}$  and  $K_{\max}$  exist such that, if  $\|x(t_0)\| < x_{\max}$ , and  $\sqrt{V(t_0)} < K_{\max}$ , then  $x$  is bounded and:

$$|e(t)| = O\left[\sup_{\tau \leq t} \Delta u(\tau)\right].$$

In addition recent results on nonlinear parameterizations allow us to extend the scope of stable adaptation to nonlinear error models of the form

$$\dot{e} = A_m e + B_p \Lambda \left( \sum_{i=1}^N g_i(x, r, \Delta u, \tilde{f}, \tilde{K}_i) \right),$$

where  $g_i$  are known, possibly nonlinear, functions of their arguments. The controller was tested on a simulation of a large four engine aircraft with four different failure scenarios. Simulations demonstrated that the proposed controller can (i) achieve stable reconfiguration following actuator failures, (ii) provide stable adaptation in the presence of significant actuator saturation, and (iii) adaptively perform control allocation for reconfiguration.

### 3. List of Publications, Inventions, and Personnel

1. C.Cao, and A.Annaswamy, "A Hierarchical Discretized-parameter Polynomial Adaptive Estimator for Nonlinearly Parameterized Systems, Technical Report 2501, Active-adaptive Control Laboratory, MIT, *International Journal of Control*, 2005 (provisionally accepted; under revision).
2. M. Schwager, A. M. Annaswamy, and E. Lavretsky, "Adaptation-Based Reconfiguration in the Presence of Actuator Failures and Saturation," *Proc. American Control Conference*, Portland, OR, 2005.
3. C.Cao, A.Annaswamy, and N. Hovakimyan "A Polynomial Adaptive Controller for Discretely Parameterized Systems", *Proc. American Control Conference*, Portland, OR, 2005.
4. C.Cao, and A.Annaswamy, "A Polynomial Adaptive Estimator for Nonlinearly Parameterized Systems", *Proceedings of American Control Conference*, pp 584-589, Boston, MA, 2004.
5. C.Cao and A.Annaswamy, "A Dead-zone Based Adaptive Filter for Systems with Unknown Parameters", *Proceedings of American Control Conference*, pp 5400-5405, Boston, MA, 2004.
6. C.Cao, and A.Annaswamy "A Polynomial Adaptive Controller for Systems with Unknown Time-delays", Technical Report 2502, Active-adaptive Control Laboratory, MIT (to be submitted to *Automatica*), 2005.
7. C.Cao, and A.Annaswamy "A Polynomial Adaptive Controller for Nonlinearly Parameterized Systems", Technical Report 2503, Active-adaptive Control Laboratory, MIT, (to be submitted to *IEEE Transactions on Automatic Control*), 2005.

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1. C.Cao, A.Annaswamy, and N. Hovakimyan "A Polynomial Adaptive Controller for Discretely Parameterized Systems", *Proc. American Control Conference*, Portland, OR, 2005.
2. C.Cao, and A.Annaswamy, "A Polynomial Adaptive Estimator for Nonlinearly Parameterized Systems", *Proceedings of American Control Conference*, pp 584-589, Boston, MA, 2004.
3. C.Cao and A.Annaswamy, "A Dead-zone Based Adaptive Filter for Systems with Unknown Parameters", *Proceedings of American Control Conference*, pp 5400-5405, Boston, MA, 2004.

**Manuscripts submitted:**

1. C. Cao and A. Annaswamy, "A hierarchical discretized-parameter polynomial adaptive estimator for nonlinearly parameterized systems," submitted to *Int. J. of Control*.

**4. List of all participating scientific personnel**

Anuradha Annaswamy (PI)

Chengyu Cao (Ph.D, Department of ME, MIT, June 2004)

**5. Report of inventions**

none

## Appendix I

### Approach (A1): The Lyapunov-Krasavoskii Functional

Assuming that  $\hat{\tau}(t) > \bar{\tau} = \tau_1 - \tau_2$  for all  $t$ , it can be shown that the time-derivative of  $V$  in section 2.1 is given by

$$\begin{aligned} \dot{V} \leq & -\alpha y^2(t) + \left[ -k y(t - \hat{\tau}(t)) y(t) + \dot{\hat{\tau}} \right] \\ & + y^2(t) \left[ \frac{\tau_1 \alpha^2}{2} + \frac{\tau_2 \alpha k}{2} + \tau_1 \varepsilon_1 \right] + y^2(t - \hat{\tau}(t)) \left[ \frac{\tau_2 k^2}{2} + \frac{\tau_1 \alpha k}{2} + \tau_2 \varepsilon_1 \right] \\ & \int_{t-\tau_1}^t \left[ \frac{\alpha^2}{2} + \frac{\alpha k}{2} - \varepsilon_1 \right] y^2(\theta) d\theta + \int_{t-\tau_2}^t \left[ \frac{k^2}{2} + \frac{\alpha k}{2} - \varepsilon_2 \right] y^2(\theta - \hat{\tau}(\theta)) d\theta. \end{aligned}$$

Choosing  $\varepsilon_1 = \frac{\alpha^2}{2} + \frac{\alpha k}{2}$ , and  $\varepsilon_2 = \frac{k^2}{2} + \frac{\alpha k}{2}$ , and defining

$$\gamma_{11} = \alpha^2 + \frac{\alpha k}{2}, \gamma_{12} = \frac{\alpha k}{2}, \gamma_{21} = \gamma_{12}, \text{ and}$$

$$\gamma_{22} = k^2 + \frac{\alpha k}{2}, \text{ we obtain that}$$

$$\dot{V} \leq -(\alpha - \gamma_{11}\tau_1 - \gamma_{12}\tau_2)y^2(t) - (\beta^2 - \gamma_{21}\tau_1 - \gamma_{22}\tau_2)y^2(t - \hat{\tau}(t)),$$

which proves that for all  $\tau_1, \tau_2 \in [0, \tau_{\max}]$ ,  $\dot{V} \leq 0$ , where

$$\tau_{\max} = \max \left\{ \alpha - \gamma_{11}\tau_1 - \gamma_{12}\tau_2 > \alpha_0, \beta^2 - \gamma_{21}\tau_1 - \gamma_{22}\tau_2 > \beta_0 \right\}$$

and  $\alpha_0, \beta_0$  are positive constants. Hence,  $V(t)$  is decreasing, and  $C|z(t)| \leq \sqrt{V(t)} \leq \sqrt{V(0)}$ ,

where  $C$  is a constant. This implies that

$$|y(t)| \leq \frac{\sqrt{V(0)}}{C} + \alpha \int_{t-\tau_1}^t |y(\theta)| d\theta + k \int_{t-\tau_2}^t |y(\theta - \hat{\tau}(\theta))| d\theta.$$

$$|y(t)| \leq \frac{\sqrt{V(0)}}{C} + \alpha \sup_{\theta \in [t-\tau_1, t]} |y(\theta)| + k \sup_{\theta \in [t-\tau_2-\tilde{\tau}, t]} |y(\theta)|, \text{ where } \tilde{\tau} = \sup_{\theta \in [t-\tau_2, t]} \hat{\tau}(\theta).$$

Therefore,

$$(1 - \alpha\tau_1 - k\tau_2) \sup_{\theta \in [t-\tau_2-\tilde{\tau}, t]} |y(\theta)| \leq \frac{\sqrt{V(0)}}{C}.$$

If we further assume that  $\alpha\tau_1 + k\tau_2 < 1$ , we may show  $|y(t)|$  is uniformly bounded. Then,  $|u(t)|$  is uniformly bounded, and  $|\dot{y}(t)|$  is uniformly bounded. Hence,  $y(t)$  is uniformly continuous.

Furthermore, it can be shown that  $y(t) \in L_2([0, \infty))$ . Hence,  $\lim_{t \rightarrow \infty} y(t) = 0$  by applying Barbalat's lemma.

We have also investigated a second class of systems with unknown time-delay in the control input ( $\tau = \tau_2 > \tau_1 = 0$ ), which is of the form,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t - \tau), \\ u(t) &= -k^T x(t - \hat{\tau}(t)), \text{ and} \\ \dot{\hat{\tau}}(t) &= -2x(t)^T Pbk^T (x(t) - x(t - \hat{\tau}(t))) - \beta^2 \|x(t - \hat{\tau}(t))\|^2, \end{aligned}$$

where  $A - bk^T$  is Hurwitz.

Then,  $\forall \alpha > 0, \exists P = P^T > 0$ , s.t.  $\alpha I = -((A - bk^T)^T P + P(A - bk^T)) > 0$ .

The stability of the adaptive controller was shown by using a Lyapunov-Krasovskii functional  $V$ ,

$$V(t) = z(t)^T P z(t) + |\hat{\tau}(t)| + \varepsilon \int_{-\tau}^0 \int_{t+\theta}^t \|x(\xi - \hat{\tau}(\xi))\|^2 d\xi d\theta,$$

$$\text{where } z(t) = x(t) - bk^T \int_{t-\tau}^t x(\theta - \hat{\tau}(\theta)) d\theta.$$

Assuming  $\hat{\tau}(t) > 0$  for all time, we obtain the time-derivative of  $V$  as follows.

$$\begin{aligned} \dot{V}(t) &= 2z(t)^T P \dot{z}(t) + \dot{\hat{\tau}}(t) + \varepsilon \int_{-\tau}^0 \frac{d}{dt} \left( \int_{t+\theta}^t \|x(\xi - \hat{\tau}(\xi))\|^2 d\xi \right) d\theta \\ &= 2x(t)^T PA x(t) - 2x(t)^T Pbk^T x(t - \hat{\tau}(t)) - 2x(t)^T A^T Pbk^T \int_{t-\tau}^t x(\theta - \hat{\tau}(\theta)) d\theta \\ &\quad + 2x(t - \hat{\tau}(t))^T kb^T Pbk^T \int_{t-\tau}^t x(\theta - \hat{\tau}(\theta)) d\theta + \dot{\hat{\tau}}(t) + \varepsilon \tau \|x(t - \hat{\tau}(t))\|^2 \\ &\quad - \varepsilon \int_{t-\tau}^t x(\theta - \hat{\tau}(\theta)) d\theta. \end{aligned}$$

$$\begin{aligned} &\leq -(\alpha - \tau \|A^T P b\|^2) \|x(t)\|^2 + \dot{\hat{\tau}}(t) + 2x(t)^T P b k^T (x(t) - x(t - \hat{\tau}(t))) \\ &\quad + \tau (\varepsilon - \|k b^T P b\|^2) \|x(t - \hat{\tau}(t))\|^2 - \left( \varepsilon - 2\|k\|^2 \right) \int_{t-\tau}^t \|x(\theta - \hat{\tau}(\theta))\|^2 d\theta. \end{aligned}$$

Using the adaptation law for  $\hat{\tau}(t)$  and setting  $\varepsilon = 2\|k\|^2$ , we have

$$\dot{V}(t) \leq -(\alpha - \tau \|A^T P b\|^2) \|x(t)\|^2 - (\beta^2 - \tau(2\|k\|^2 - \|k b^T P b\|^2)) \|x(t - \hat{\tau}(t))\|^2.$$

For small enough  $\tau$ , we may take  $\alpha$  and  $\beta$ , s.t.

$$\alpha - \tau \|A^T P b\|^2 > 0, \text{ and } \beta^2 - \tau(2\|k\|^2 - \|k b^T P b\|^2) > 0.$$

Then,  $\dot{V}(t) \leq 0$ .

Also,  $V(t)$  is decreasing, hence  $C\|z(t)\| \leq \sqrt{V(t)} \leq \sqrt{V(0)}$ , where  $C$  is a constant. This implies that

$$\|x(t)\| \leq \frac{\sqrt{V(0)}}{C} + \tau \|b k^T\| \int_{t-\tau}^t \|x(\theta - \hat{\tau}(\theta))\| d\theta, \text{ where } \|b k^T\| = \inf_{\|x\|>0} \left( \frac{\|b k^T x\|}{\|x\|} \right).$$

Therefore,

$$\|x(t)\| \leq \frac{\sqrt{V(0)}}{C} + \tau \|b k^T\| \sup_{\theta \in [t-\tau-\tilde{\tau}, t]} \|x(\theta)\|, \text{ where } \tilde{\tau} = \sup_{\theta \in [t-\tau, t]} \hat{\tau}(\theta).$$

$$(1 - \tau \|b k^T\|) \sup_{\theta \in [t-\tau-\tilde{\tau}, t]} \|x(\theta)\| \leq \frac{\sqrt{V(0)}}{C}.$$

If we further assume  $1 - \tau \|b k^T\| > 0$ , we may show  $\|x(t)\|$  is uniformly bounded. Then,  $\|u(t)\|$  is uniformly bounded, and  $\|\dot{x}(t)\|$  is uniformly bounded. Hence,  $\|x(t)\|$  is uniformly continuous.

Furthermore, it can be shown that  $\|x(t)\| \in L_2([0, \infty))$ . Hence,  $\lim_{t \rightarrow \infty} \|x(t)\| = 0$  by applying

Barbalat's lemma.

We note that in the second class of systems, the control input does not require any knowledge of  $\tau$  and hence does not require adaptation. Currently, we are examining those systems that explicitly require knowledge of the time-delay in the control input.

**Approach (A2) Treating Time-delay as a nonlinear parameter**

Please see the appendices A2-1.pdf, A2-2.pdf, and A2-3.pdf for a summary of the results obtained in this category.

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